S181. Proposed by Titu Andreescu, University of Texas at Dallas, USA.

Let a and b be positive real numbers such that

$$|a-2b| \le \frac{1}{\sqrt{a}} \text{ and } |2a-b| \le \frac{1}{\sqrt{b}}.$$

Prove that $a+b \le 2$.

Solution by Arkady Alt , San Jose ,California, USA.

Since $\begin{cases} |a-2b| \le \dfrac{1}{\sqrt{a}} \\ |2a-b| \le \dfrac{1}{\sqrt{b}} \end{cases} \Leftrightarrow \begin{cases} a^2-4ab+4b^2 \le \dfrac{1}{a} \\ 4a^2-4ab+b^2 \le \dfrac{1}{b} \end{cases} \Leftrightarrow$

$\begin{cases} a^3-4a^2b+4ab^2 \le 1 \\ 4a^2b-4ab^2+b^3 \le 1 \end{cases}$ then $2 \ge a^3-4a^2b+4ab^2+4a^2b-4ab^2+b^3 =$

a^3+b^3 and, therefore, by Power Mean Inequality we have

$\left(\dfrac{a+b}{2}\right)^3 \le \dfrac{a^3+b^3}{2} \le 1 \Rightarrow \dfrac{a+b}{2} \le 1 \Leftrightarrow a+b \le 2.$